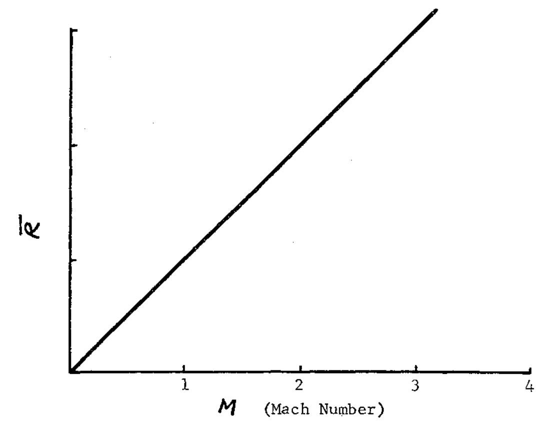

Fig. 2 $\bar{R}$ vs Mach number.

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Shock-Layer Radiation for Sphere-Cones with Radiative Decay

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INTERPLANETARY vehicles re-entering the earth's atmosphere at hyperbolic velocities will be subjected to severe thermal environments. For the extreme re-entry velocities presently being considered in missions analysis, radiation is a major heat-transfer mechanism. Consequently, slender vehicle configurations, such as spherically blunted cones, are attractive.

Manned vehicles may re-enter at velocities in excess of 60,000 fps. Trajectory requirements dictate descent to altitudes of approximately 200,000 ft. At these conditions, the effects of radiative decay in the shock layer become important. Previous investigations of these effects for detached shock layers are reported in Refs. 1–5. The results of these references illustrate the nonadiabatic nature of the shock layer at very high entry velocities. In this note, approximate methods for determination of the effect of radiative decay on the shock-layer radiation toward sphere-cones will be presented. As pointed out in Ref. 5, for most conditions of interest, the effect of self-absorption in the shock layer is small.

As the gas particles travel downstream from the shock wave, the enthalpy decays as a result of radiation loss; that is,

$$-u(dh/ds) = 4k\sigma T^4 \tag{1}$$

where u is the particle velocity, s the distance along the streamline, k the Planck-mean mass absorption coefficient, σ the Stefan-Boltzmann constant, and T the absolute temperature. The change of the particle kinetic energy has been neglected in the energy balance.

For calculation of radiation to the body, the shock layer is approximated by a semi-infinite plane-parallel slab having the local shock-layer thickness and radiation distribution. Then,

$$q = \int_0^{\Delta} 2\rho k\sigma T^4 dy \tag{2}$$

where q is radiation toward body, ρ local density with radiative decay, Δ shock-layer thickness, and y distance from body. Introducing the Howarth-Dorodnitsyn variable $y_a = \int_0(\rho/\rho_s)dy$, Eq. (2) may be written as

$$q = \int_0^{\Delta_a} 2\rho_s k\sigma T^4 dy_a \tag{3}$$

where ρ_s is the density immediately behind shock, and Δ_a is the adiabatic shock-layer thickness. Equation (3) may be transformed to yield

$$\frac{q}{q_a} = \int_0^1 \left(\frac{kT^4}{k_s T_s^4}\right) d\left(\frac{y_a}{\Delta_a}\right) \tag{4}$$

where $q_a \equiv 2\rho_s k_s \sigma T_s^4 \Delta_a$, radiation toward the body from the "adiabatic" shock layer, and where k_s, T_s correspond to conditions immediately behind the shock.

The adiabatic shock-layer thickness may be obtained from the constant-density solutions. For a sphere (Ref. 6, p. 160), this thickness in terms of body radius R is

$$\Delta_a/R = \epsilon/[1 + (\tfrac{8}{3}\epsilon)^{1/2} - \epsilon] \tag{5}$$

where $\epsilon = \rho_\infty/\rho_s$ and ρ_∞ = freestream density. For a cone (Ref. 6, p. 146),

$$\Delta_a/S = \tfrac{1}{2}\epsilon \tan\theta_s \approx \tfrac{1}{2}\epsilon \tan\theta_c \tag{6}$$

where S is the distance from stagnation point along the conical surface, θ_s shock angle, and θ_c cone half-angle.

The particle velocity along the stagnation streamline in the transformed coordinate system is approximated as that given by the incompressible potential flow expression:

$$u = (\rho_\infty u_\infty/\rho)(y_a/\Delta_a) \tag{7}$$

where u_∞ is the freestream velocity. Interpreting $-ds = dy$, Eqs. (1) and (7) may be combined to yield

$$d\ln(y_a/\Delta_a) = (1/\Gamma_s)(k_s T_s^4/kT^4)d(h/h_s) \tag{8}$$

where

$$\Gamma_s \equiv 2q_a/\rho_\infty u_\infty h_s \approx 4q_a/\rho_\infty u_\infty^3$$

and h_s is the enthalpy immediately behind the shock wave.

For the conical body, the local shock layer consists of approximately parallel streamlines that emanate from different positions along the conical shock wave. From a continuity consideration,

$$2\pi(S\sin\theta_c)(u_\infty\cos\theta_s)\rho dy = \pi\rho_\infty u_\infty d[(S - x)\sin\theta_c]^2$$

or, with $\theta_c \approx \theta_s$,

$$\rho dy = -\rho_\infty[1 - (x/S)]\tan\theta_s dx \tag{9}$$

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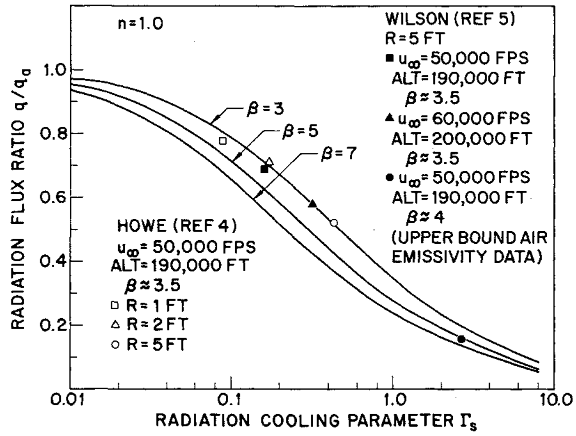

Fig. 1 Effect of radiative decay on stagnation radiative flux.

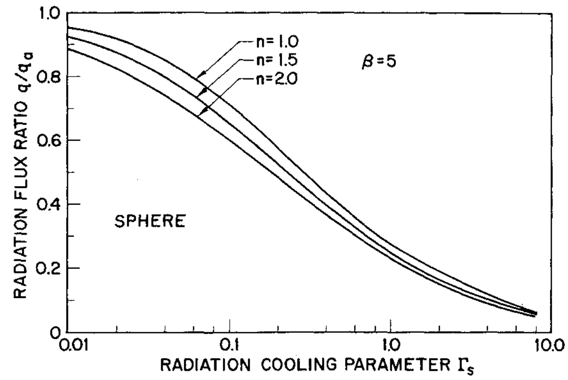

Fig. 3 Effect of velocity distribution assumption on decay correlation.

where x is the distance traversed in the shock layer by the gas particle at position (S,y). The shock layer is assumed thin, so that the particle velocity is approximately $u_\infty \cos\theta_s$. With $ds = dx$, and using Eqs. (1) and (6),

$$d(x/S) = -(1/2\Gamma_c)(k_sT_s^4/kT^4)d(h/h_s) \tag{10}$$

where

$$\Gamma_c \equiv 2q_a/\rho_\infty(u_\infty \sin\theta_s)h_s \approx 4q_a/\rho_\infty(u_\infty \sin\theta_s)^3$$

Using the Howarth-Dorodnitsyn variable, Eq. (11) may be easily obtained from Eqs. (6) and (9):

$$y_a/\Delta_a = [1 - (x/S)]^2 \tag{11}$$

Therefore, from Eqs. (10) and (11),

$$d(y_a/\Delta_a)^{1/2} = (1/2\Gamma_c)(k_sT_s^4/kT^4)d(h/h_s) \tag{12}$$

Once the h/h_s distribution is known, the $kT^4/k_sT_s^4$ distribution may be calculated and then Eq. (4) integrated to yield q/q_a. For a given set of thermodynamic and radiative properties (e.g., Ref. 7), q/q_a will be a function of Γ_s or Γ_c. Both Γ_s and Γ_c can be identified as the ratio of total radiation loss from an isothermal shock layer to the normal enthalpy flux immediately behind the shock wave.

For hypervelocity re-entry, the pressure across the shock layer is approximately constant. The ratio $kT^4/k_sT_s^4$ may then be expressed as a function of h/h_s. Integration of Eqs. (8) and (12) gives the h/h_s distribution across the shock layer.

For a first-order radiation calculation, the following correlations are useful:

$$\left(\frac{\partial \ln kT^4}{\partial \ln h}\right)_p \equiv \beta \approx \text{const} \qquad \left(\frac{\partial \ln\rho}{\partial \ln h}\right)_p \approx -1 \tag{13}$$

The partial differentiations are for conditions of constant

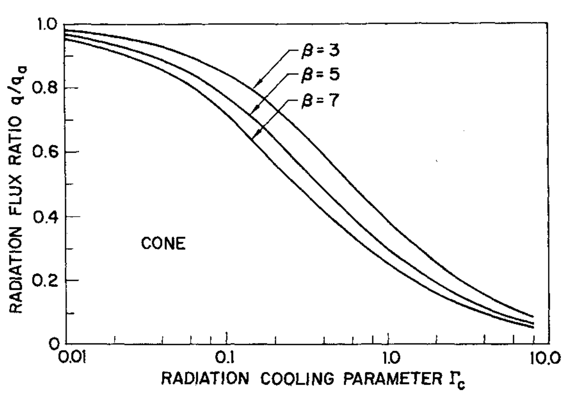

Fig. 2 Effect of radiative decay on conical body radiative flux.

pressure. Isothermal shock-layer radiation data frequently are presented at constant densities instead of at constant pressures. When this is the case, the following identity may be used to determine β:

$$\left(\frac{\partial \ln kT^4}{\partial \ln h}\right)_p \equiv \left(\frac{\partial \ln \rho kT^4}{\partial \ln h}\right)_\rho + \left(\frac{\partial \ln\rho}{\partial \ln h}\right)_p \left[\left(\frac{\partial \ln\rho kT^4}{\partial \ln\rho}\right)_h - 1\right] \tag{14}$$

With a constant β, the h/h_s distribution may be obtained from Eqs. (8) and (12) and q/q_a from Eq. (4). Let $\eta = y_a/\Delta_a$. Then, for a sphere,

$$h/h_s = [1 - \Gamma_s(\beta - 1)\ln\eta]^{-1/(\beta-1)} \tag{15}$$

$$\frac{q}{q_a} = \int_0^1 [1 - \Gamma_s(\beta - 1)\ln\eta]^{-\beta/(\beta-1)}\, d\eta \tag{16}$$

for a cone,

$$h/h_s = [1 + 2\Gamma_c(\beta - 1)(1 - \eta^{1/2})]^{-1/(\beta-1)} \tag{17}$$

$$\frac{q}{q_a} = \frac{1}{2\,\Gamma_c^2(\beta - 1)}\left[\frac{1}{\beta - 2} + g - \left(\frac{\beta - 1}{\beta - 2}\right) g^{(\beta-2)/(\beta-1)}\right] \tag{18}$$

where $g = 1 + 2\Gamma_c(\beta - 1)$.

Figures 1 and 2 show q/q_a vs Γ_s and Γ_c, respectively, with β as a parameter. The results of Refs. 4 and 5 are also shown in Fig. 1, with the approximate values of β noted. The comparison illustrates that the relatively simple first-order radiation calculation can give adequate results for engineering design purposes.

For the sphere, if the velocity distribution along the stagnation streamline is not linear as given by Eq. (7) but is given by $u = \rho_\infty u_\infty \eta^n/\rho$, the enthalpy distribution and q/q_a can be easily shown to be

$$\frac{h}{h_s} = \left[1 + \frac{\Gamma_s(\beta - 1)}{(n - 1)}(\eta^{1-n} - 1)\right]^{-1/(\beta-1)} \tag{19}$$

$$\frac{q}{q_a} = \int_0^1 \left[1 + \frac{\Gamma_s(\beta - 1)}{(n - 1)}(\eta^{1-n} - 1)\right]^{-\beta/(\beta-1)} d\eta \tag{20}$$

Figure 3 compares the results based on the velocity distributions with $n = 1$, 1.5, and 2. It is seen that q/q_a is not a very strong function of the velocity distribution.

For sphere-cones, the pure-cone solution given by Eqs. (17) and (18) should be good for large values of S/R. For values of S/R in the order of unity, the effect of the blunt nose is significant. However, for a sphere, according to the results of Ref. 5, the effect of radiative decay on the ratio of the local radiative flux to stagnation radiative flux is relatively small. This fact and the first-order radiation calculation enable the shock-layer radiation for sphere-cones to be estimated without difficulty.

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Electron Fluctuations in an Equilibrium Turbulent Plasma

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I. Introduction

THE radar cross section of an underdense turbulent wake depends on the mean square of the electron density fluctuations and on their spectral distribution.[1] Since no experiment indicating the fluctuation magnitude is available at this writing, one is forced to resort to empirical descriptions of the turbulent mixing process. One such empirical model, variously known as the "marble-cake" or "inviscid convection" model, has been elaborated upon by Feldman and Proudian[2] and by Lin and Hayes.[3] A detailed criticism of this pseudoturbulent process is permitted here neither by the limited space nor by our long-lacking understanding of the problem even when ionization is absent.

Instead of attempting to predict the magnitude of the gasdynamical (temperature, density, etc.) fluctuations, we will attempt to simply derive formulas connecting these gasdynamical fluctuations to electron density fluctuations in the extreme of equilibrium flow. Here, the "equilibrium" terminology refers to the microscopic processes enacted within the details of the turbulent structure. In this manner, we limit the present formulation to the case where the local electron density at a point is determined only by the gas density and temperature at that point.

Implicit in the following work is the assumption that, although the turbulence affects the electron population through density or temperature fluctuations, the electrical, diffusive, or inertial effects of the electrons and ions do not affect turbulent mixing as presently, albeit sketchily, understood. In this way, the results of the present paper can be applied to experimental findings on gasdynamical fluctuations alone, and the corresponding electron fluctuation magnitude can be inferred from those. Another purpose of the present formulation is to provide relations useful to any type of turbulent plasma rather than an ionized wake alone, without specifying the mixing model as in the case of Feldman[2] and Lin.[3]

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II. Interdependence of Gasdynamical Fluctuations

Since electron density fluctuations are related to the gasdynamical fluctuations, we discuss first the possibility of describing the turbulent field by means of the fluctuations of only one dynamic or thermodynamic variable.

Many years ago Kovasznay[4] pointed out that the gasdynamical fluctuations in a turbulent gas can be uniquely decomposed into three linearly independent fluctuation "modes": entropy, sound, and vorticity. In each of these modes there exists a unique relationship among pressure, density, temperature, and velocity fluctuations as dictated by the equations of conservation. In the so-called entropy mode, the temperature T and density ρ fluctuations are related by

$$\Delta\rho/\bar{\rho} = -(\Delta T/\bar{T}) \qquad (1)$$

where bars denote averages and Δ stands for the fluctuation. In the "sound" mode, the corresponding relationship is

$$\Delta\rho/\bar{\rho} = [1/(\gamma - 1)](\Delta T/\bar{T}) \qquad (2)$$

where γ is the ratio of specific heats. The "vorticity" mode involves fluctuations in the fluid velocity only and is, therefore, of no consequence as regards the electron density.

Physical considerations usually dictate the fluctuation mode (sound or entropy) predominant to a given situation.† Working with turbulent supersonic boundary layers, Kovasznay[4] and later Kistler[5] found that, in the absence of considerable sound radiation, the sound mode seems negligible compared to the entropy mode. In a turbulent hypersonic wake, such "sound generators" often are the shocklet-capped "turbs" found in the near wake, whose mean speed is considerably lower than that of the external flow; in the far wake, by contrast, the wake has acquired the velocity of the external stream and "sound" production is negligible, and the persisting wake temperatures probably make the vorticity and entropy modes dominant. Insofar as the electron fluctuations are concerned, the latter mode would then be the only one of significance.

III. Average Electron Density

At equilibrium, the instantaneous electron number density n_e is given in terms of the gas number density n and temperature T by the Saha relation

$$n_e^2 = AnT^{3/2}e^{-(B/T)} \qquad (3)$$

where A is a constant. The quantity $B \equiv E_i/k$, where E_i is the ionization energy and k the Boltzmann constant.

Suppose that the time dependence of the turbulent gas temperature can be written as

$$T(t) = \bar{T} + \Delta T(t) \qquad (4)$$

where the time average $\overline{\Delta T}$ is zero by definition, so that $\bar{T}$ is the average (mean) gas temperature. From (3), we get

$$n_e^2(\bar{T}) = A\bar{n}\bar{T}^{3/2}e^{-(B/\bar{T})} \qquad (5)$$

We will first show that $n_e(\bar{T})$ is not, in fact, the average electron density which obtains as the electron density fluctuates.

We show this as follows: if $n_e(T)$ is expanded around $\bar{T}$, we obtain

$$n_e(T) = n_e(\bar{T}) + \left(\frac{\partial n_e}{\partial T}\right)_{\bar{T}} \Delta T + \frac{1}{2}\left(\frac{\partial^2 n_e}{\partial T^2}\right)_{\bar{T}} (\Delta T)^2 + \ldots + \frac{1}{m!}\left(\frac{\partial^m n_e}{\partial T^m}\right)_{\bar{T}} (\Delta T)^m + \left(\frac{\partial n_e}{\partial \rho}\right)_{\bar{\rho}} \Delta\rho + \ldots \qquad (6)$$

† In the entropy mode, temperature and density fluctuations are perfectly anticorrelated [Eq. (1)], and in the sound mode, they are perfectly correlated [Eq. (2)]. If feasible, a correlation experiment could therefore resolve this point.